Advances in tractable methodologies to solve optimal power flow in transmission and distribution systems

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OUTLINE

- Motivation

- Procurement of ancillary services (congestion & voltage control) by DSO and TSO in **day-ahead operation planning**

- **DSO** level: stochastic multi-period AC optimal power flow (S-MP-OPF) problem
  - **Tailored** tractable solution algorithm through sequential linear approximations
  - Numerical performance
  - Research work led by Muhammad Usman (LIST)

- **TSO** level: stochastic multi-period AC **security-constrained** optimal power flow (S-MP-SCOPF) problem
  - **Tailored** tractable solution algorithm through (**different!**) sequential linear approximations
  - Numerical performance
  - Research work led by Mohammad Iman Alizadeh (LIST)

- Conclusions and future work

POWER SYSTEMS OF THE FUTURE (2030+) AT BOTH DSO AND TSO LEVELS

- Characteristics:
  - High penetration of a myriad of distributed energy resources (DER), particularly **variable** renewable generation (RES)
  - Emerging **additional** flexibility options: energy storage systems and flexible “loads” (e.g. aggregated, deferable energy entities)

- RES **variability** and **hardness to predict** **undermine reliable operation** (e.g. congestion, voltage)
  - Operators need to procure energy flexibility, **including** from DER, in day-ahead and activate it (if needed) in real-time

CHALLENGES AT BOTH DSO AND TSO LEVELS

- Computation challenges in decision-making tools for DSO and TSO:
  - Uncertainty in renewables production ➡ modelling **stochasticity**
  - Energy coupling in time ➡ modelling **multiple time periods interlinked**

- The problem size is (**number of uncertainty scenarios** x **number of time-periods**) times **larger**!

- We envision major upgrades in the decision-making tools that must be able to solve:
  - At DSO level: stochastic multi-period AC optimal power flow (**S-MP-OPF**) problems
  - At TSO level: stochastic multi-period AC security-constrained optimal power flow (**S-MP-SCOPF**) problems

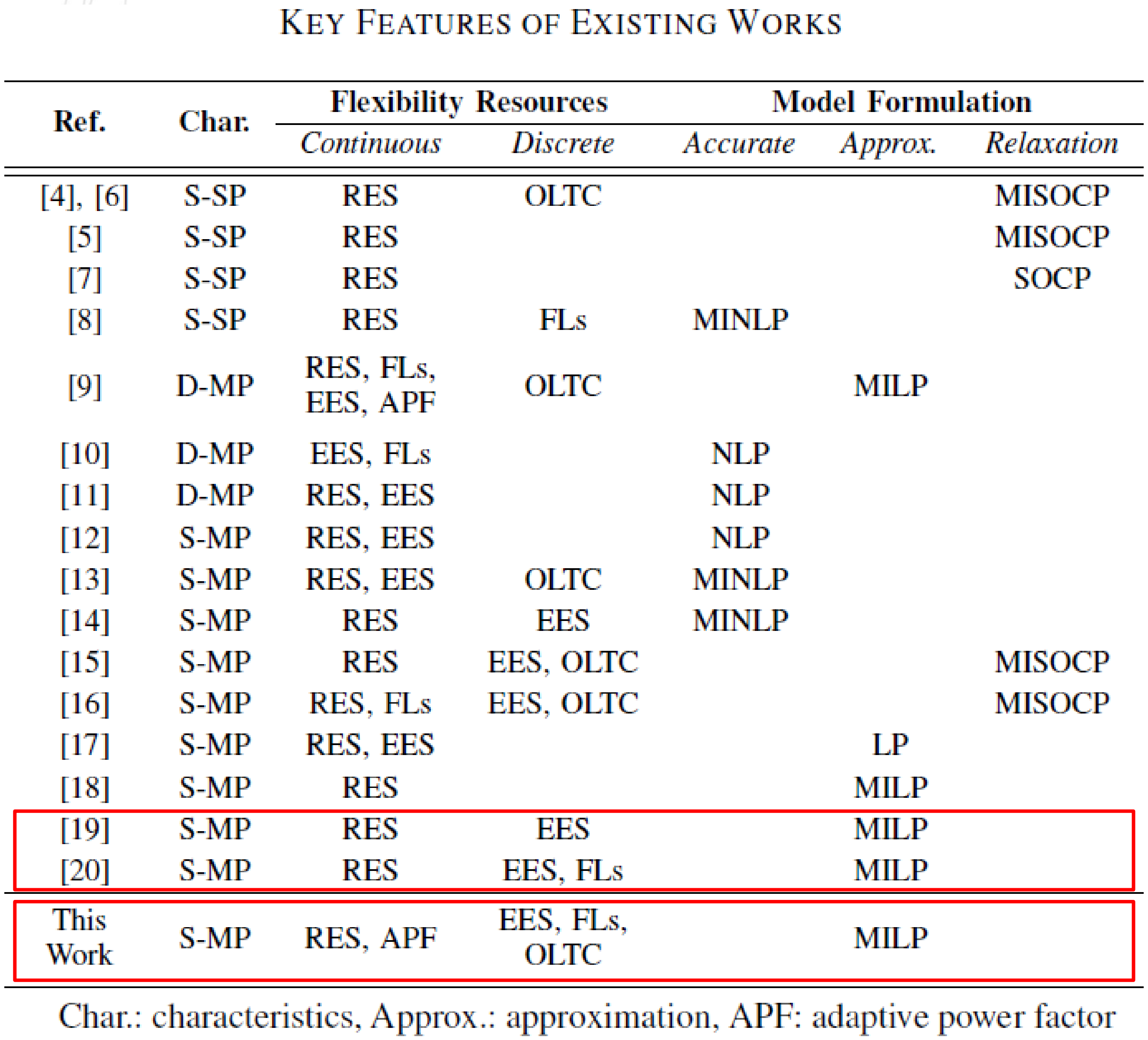
- Focus on **tractability**: **solving sequentially a limited number of linear approximations** of the full problems

- Assumptions:
  - Centralized optimization
  - At DSO level: **adequate observability** of the medium voltage (**MV)** grid and balanced operation

DSO PROBLEM

Flexibility procurement by DSOs in future smart grids including from DER through S-MP-OPF

DSO PROBLEM: SALIENT FEATURES OF EXISTING WORKS



KEY FEATURES OF EXISTING WORKS

Ref.	Char.	Flexibility Resources		Model Formulation		
		Continuous	*Discrete*	*Accurate*	*Approx.*	*Relaxation*
[4], [6]	S-SP	RES	OLTC			MISOCP
[5]	S-SP	RES				MISOCP
[7]	S-SP	RES				SOCP
[8]	S-SP	RES	FLs	MINLP		
[9]	D-MP	RES, FLs, EES, APF	OLTC		MILP	
[10]	D-MP	EES, FLs		NLP		
[11]	D-MP	RES, EES		NLP		
[12]	S-MP	RES, EES		NLP		
[13]	S-MP	RES, EES	OLTC	MINLP		
[14]	S-MP	RES	EES	MINLP		
[15]	S-MP	RES	EES, OLTC			MISOCP
[16]	S-MP	RES, FLs	EES, OLTC			MISOCP
[17]	S-MP	RES, EES			LP	
[18]	S-MP	RES			MILP	
[19]	S-MP	RES	EES		MILP	
[20]	S-MP	RES	EES, FLs		MILP	
This Work	S-MP	RES, APF	EES, FLs, OLTC		MILP	

Char.: characteristics, Approx.: approximation, APF: adaptive power factor

DSO PROBLEM: CHALLENGES

- Some flexibility options (storage, flexible loads, OLTCs) involve **binary variables**

- The problem is mixed-integer non-linear programming (**MINLP**)
  - Computationally intractable: it cannot be used for practical applications

- A novel scalable solution is proposed, which relies on:
  - A mixed integer linear programming (MILP) model
    - **Linear approximations of AC power flow equations and branch current expressions**

LINEAR APPROXIMATIONS

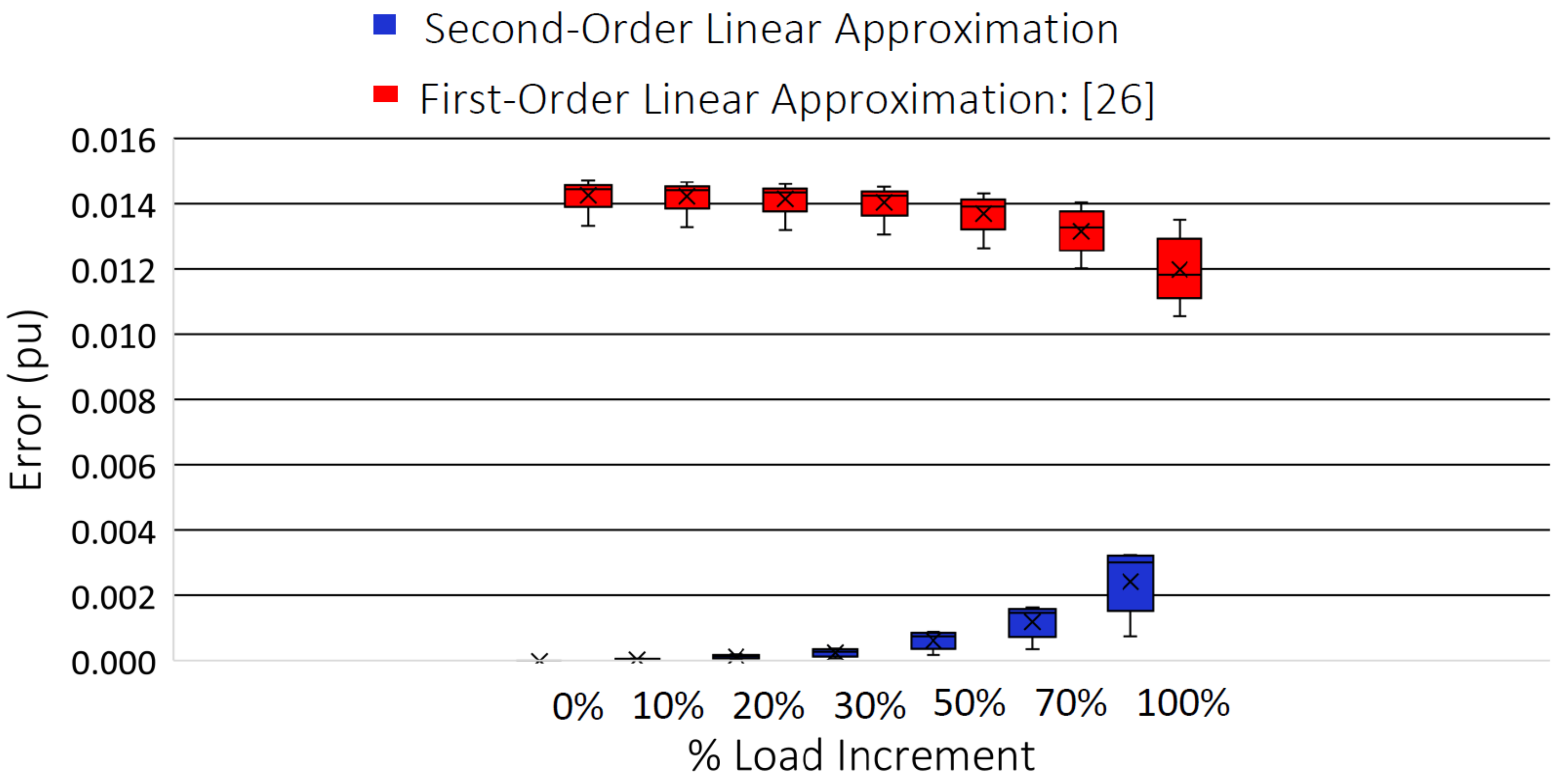
- Linear active/reactive power flows and linear longitudinal branch current expressions:

$$
\begin{aligned}
P_{ij,t} &= \alpha^{p}_{i,t}\boxed{V^2_{i,t}} + \alpha^{p}_{j,t}\boxed{V^2_{j,t}} + \beta^{p}_{ij,t}\theta_{ij,t} + \gamma^{p}_{i,t} \\
Q_{ij,t} &= \alpha^{q}_{i,t}V^2_{i,t} + \alpha^{q}_{j,t}V^2_{j,t} + \beta^{q}_{ij,t}\theta_{ij,t} + \gamma^{q}_{i,t} \\
I^2_{ij,t} &= (g^2_{ij} + b^2_{ij})(\alpha^{I}_{i,t}V^2_{i,t} + \alpha^{I}_{j,t}V^2_{j,t} + \beta^{I}_{ij,t}\theta_{ij,t} + \gamma^{I}_{i,t})
\end{aligned}
$$

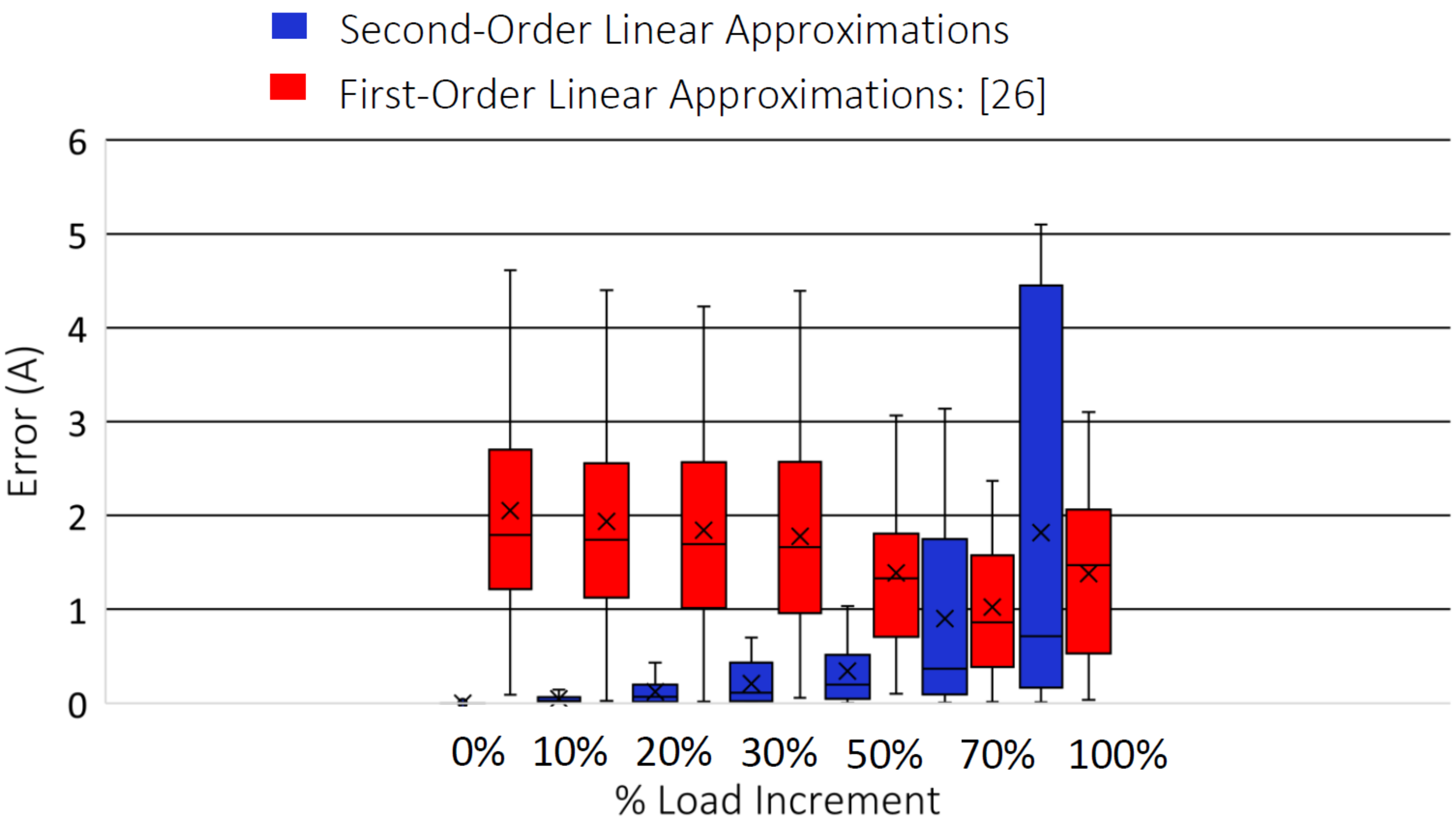
- (α, β and γ) coefficients depend upon initial point-of-linearization

- the developed linear expressions are **very accurate** because:
  - they are linear in terms of **V^2** and θ variables
  - + they include **second order terms** (Taylor expansion of trigonometric terms)

QUALITY OF LINEAR APPROXIMATIONS (BUS VOLTAGES)



QUALITY OF LINEAR APPROXIMATIONS (CURRENT IN POWER LINES)



TRACTABLE MILP MODEL

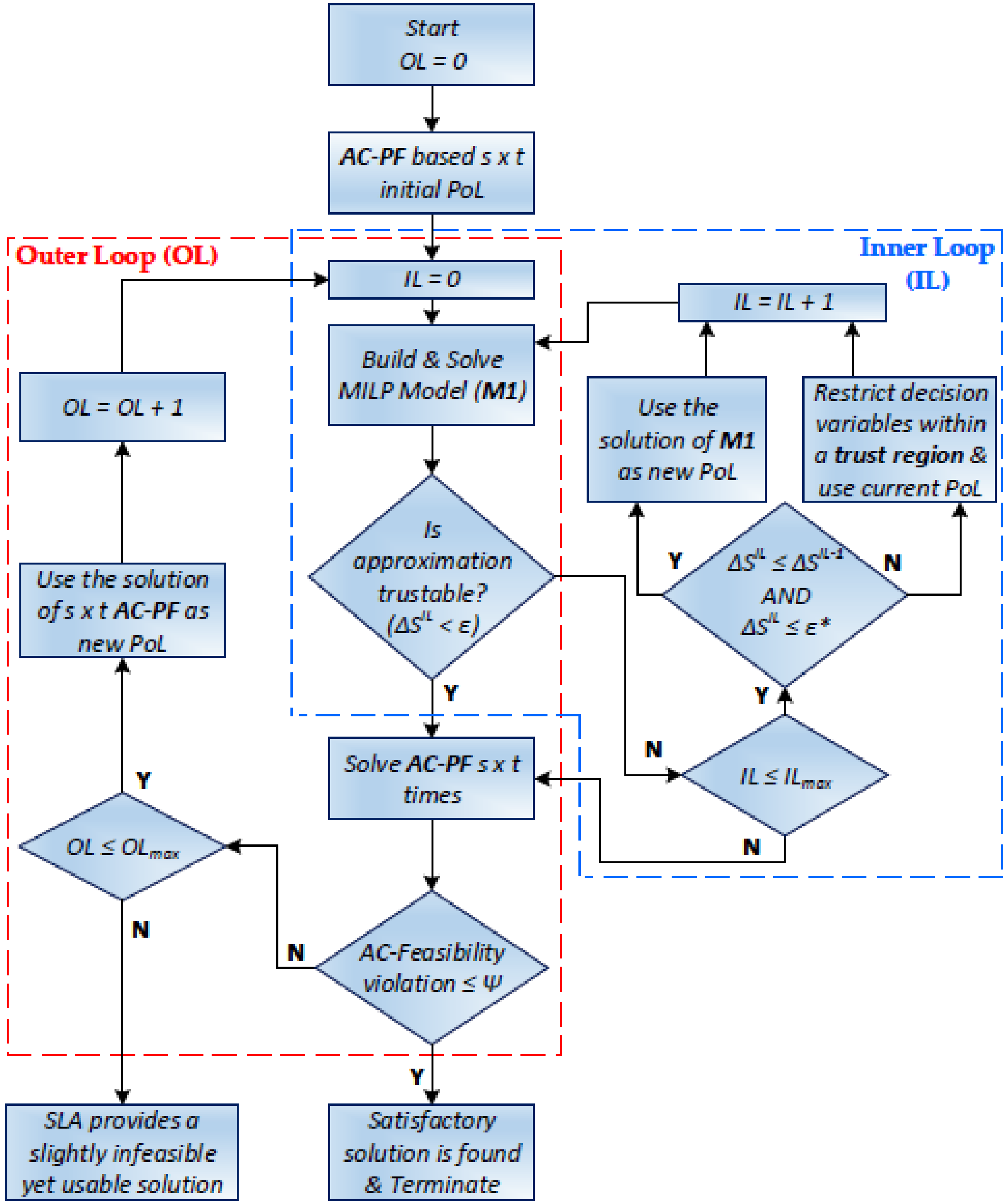
- Objective: Minimize the expected cost of DER output deviation from the market schedule

$$\min \sum_{t\in T}\left\{\sum_{i\in G} c^{curt}_{i,p} P^{curt}_{i,t} + \sum_{i\in B} c^{str}_{i,b}\left(P^{dch}_{i,t} - P^{ch}_{i,t}\right) + c^{fl}_{i,l}\sum_{i\in F}\left(P^{od}_{i,t} + P^{ud}_{i,t}\right)\right\}\Delta T + c^{oltc}_{ij}\cdot \kappa_{if,t}$$

- subject to:
  - linear constraints:
    - Linear active and reactive power balance
    - Linear branch flow loading limits
    - Active and reactive power limits on the import from the HV upstream grid
    - Node voltage magnitude limits

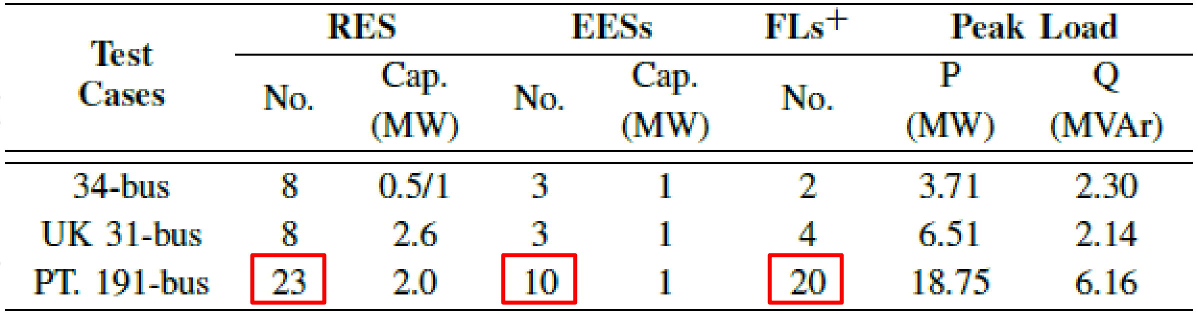
  - mixed-**integer** linear constraints:
    - Active power curtailment and reactive power provision from RES
    - Constraints modelling the behaviour of flexible loads/electrical energy storage/OLTC

SEQUENTIAL LINEARIZATION ALGORITHM (SLA)

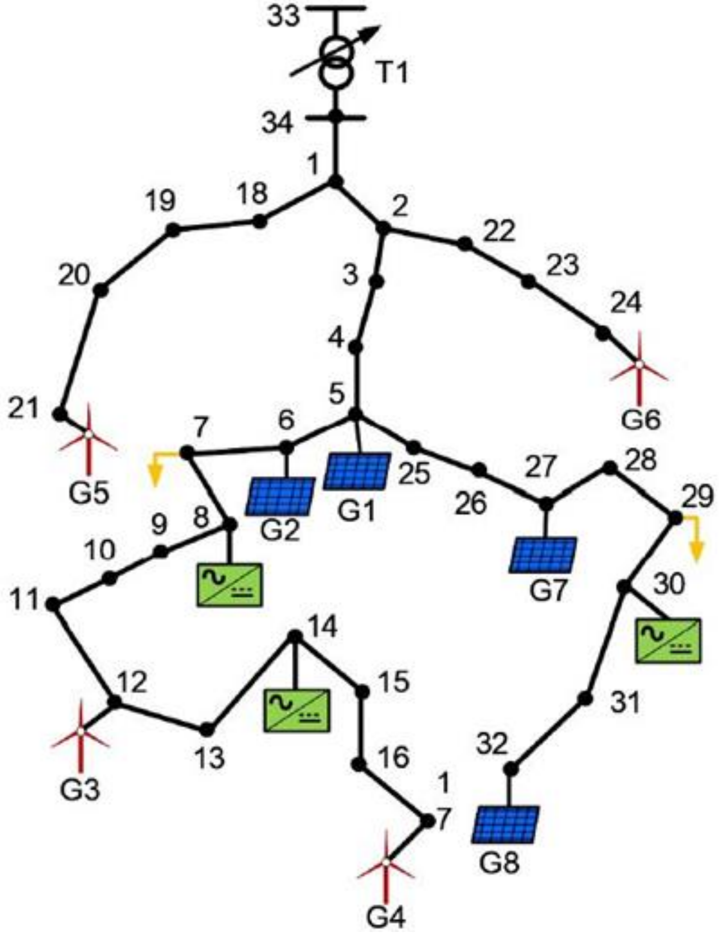


CASE STUDIES

- Proposed tractable approach is evaluated on the three distribution grids



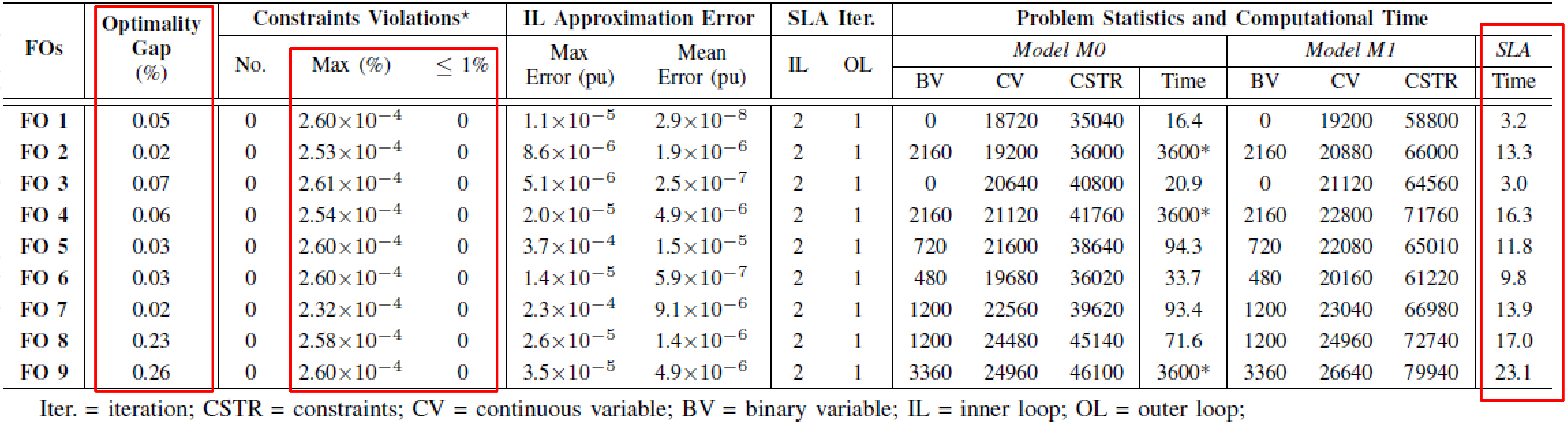
Test Cases	RES		EESs		FLs$^+$	Peak Load	
	No.	Cap. (MW)	No.	Cap. (MW)	No.	P (MW)	Q (MVAr)
34-bus	8	0.5/1	3	1	2	3.71	2.30
UK 31-bus	8	2.6	3	1	4	6.51	2.14
PT. 191-bus	23	2.0	10	1	20	18.75	6.16



- **24**-96 time periods and **10**-50 RES uncertainty scenarios (generated using ARIMA)

- All networks provide flexibility in the form of:
  - Active power curtailment of RES
  - Reactive power provision from RES
  - Active power charging/discharging of an electrical energy storage
  - Active power over/under-demand of a flexible load
  - On-load tap changing transformer

NUMERICAL PERFORMANCE ON 34-BUS GRID



FOs	Optimality Gap (%)	Constraints Violations*			IL Approximation Error		SLA Iter.		Problem Statistics and Computational Time							
									Model M0				*Model M1*			*SLA*
		No.	Max (%)	$\leq$ 1%	Max Error (pu)	Mean Error (pu)	IL	OL	BV	CV	CSTR	Time	BV	CV	CSTR	Time
FO 1	0.05	0	2.60×10^{-4}	0	1.1×10^{-5}	2.9×10^{-8}	2	1	0	18720	35040	16.4	0	19200	58800	3.2
FO 2	0.02	0	2.53×10^{-4}	0	8.6×10^{-6}	1.9×10^{-6}	2	1	2160	19200	36000	3600*	2160	20880	66000	13.3
FO 3	0.07	0	2.61×10^{-4}	0	5.1×10^{-6}	2.5×10^{-7}	2	1	0	20640	40800	20.9	0	21120	64560	3.0
FO 4	0.06	0	2.54×10^{-4}	0	2.0×10^{-5}	4.9×10^{-6}	2	1	2160	21120	41760	3600*	2160	22800	71760	16.3
FO 5	0.03	0	2.60×10^{-4}	0	3.7×10^{-4}	1.5×10^{-5}	2	1	720	21600	38640	94.3	720	22080	65010	11.8
FO 6	0.03	0	2.60×10^{-4}	0	1.4×10^{-5}	5.9×10^{-7}	2	1	480	19680	36020	33.7	480	20160	61220	9.8
FO 7	0.02	0	2.32×10^{-4}	0	2.3×10^{-4}	9.1×10^{-6}	2	1	1200	22560	39620	93.4	1200	23040	66980	13.9
FO 8	0.23	0	2.58×10^{-4}	0	2.6×10^{-5}	1.4×10^{-6}	2	1	1200	24480	45140	71.6	1200	24960	72740	17.0
FO 9	0.26	0	2.60×10^{-4}	0	3.5×10^{-5}	4.9×10^{-6}	2	1	3360	24960	46100	3600*	3360	26640	79940	23.1

Iter. = iteration; CSTR = constraints; CV = continuous variable; BV = binary variable; IL = inner loop; OL = outer loop;

CONCLUSIONS

- Extensive results demonstrate **empirically** (we don't have a mathematical proof of convergence!) that:

- regarding **solution accuracy**, the proposed SLA provides an **optimal** and **feasible** solution:
  - The optimality gap is nearly 0% (its worst value is 0.21%) in all test cases under all flexible options.
  - AC-feasible solution is obtained within few SLA iterations
    - the accuracy of developed linear approximations remains intact **under stressed operation conditions**

- regarding **computational efficiency**, the proposed SLA **reduces the solution time** by a factor of 12-30 wrt MINLP (when this converges)
  - The SLA outperforms alternative linear models
  - Furthermore, the solution time of SLA remains moderate (below one hour) for the 191-bus system under larger number of scenarios (up to 50) or time-periods (up to 96)

- Scalable (by extrapolation) to grids up to 1000 nodes but under limited number of binary variables (storage/flexible loads)

- Works well on both radial and weakly-meshed distribution grids

FOR FURTHER READING

- Problem formulation:
  - [1] M. Usman, F. Capitanescu, A stochastic multi-period AC optimal power flow for provision of flexibility services in smart grids, EEE PowerTech conference, 2021.

- Derivation of the linear expressions:
  - [2] M. Usman, F. Capitanescu, A New Second-Order Linear Approximation to AC OPF Managing Flexibility Provision in Smart Grids, IEEE SEST Conference 2021.

- Solution algorithms:
  - [3] M. Usman, F. Capitanescu, A Novel Tractable Methodology to Stochastic Multi-Period AC OPF in Active Distribution Systems, IEEE Transactions on Power Systems, in press, 2022.
  - [4] M. Usman, F. Capitanescu, Three Solution Approaches to Stochastic Multi-Period AC Optimal Power Flow in Active Distribution Systems, IEEE Transactions on Sustainable Energy, in press, 2022.

The code source of [3] will be open early next year on ATTEST repository: stay tunned!

TSO PROBLEM

- S-MP-SCOPF problem for flexibility procurement by TSO from:
    - conventional generators
    - RES
    - storage
    - flexible “loads”

- Nonlinear and non-convex constraints:
    - Nodal AC active power balance
    - Nodal AC reactive power balance
    - Thermal limits of power lines/transformers

- The problem is **approximated** as non-linear programming
    - Computationally heavy: it cannot be scaled for practical applications

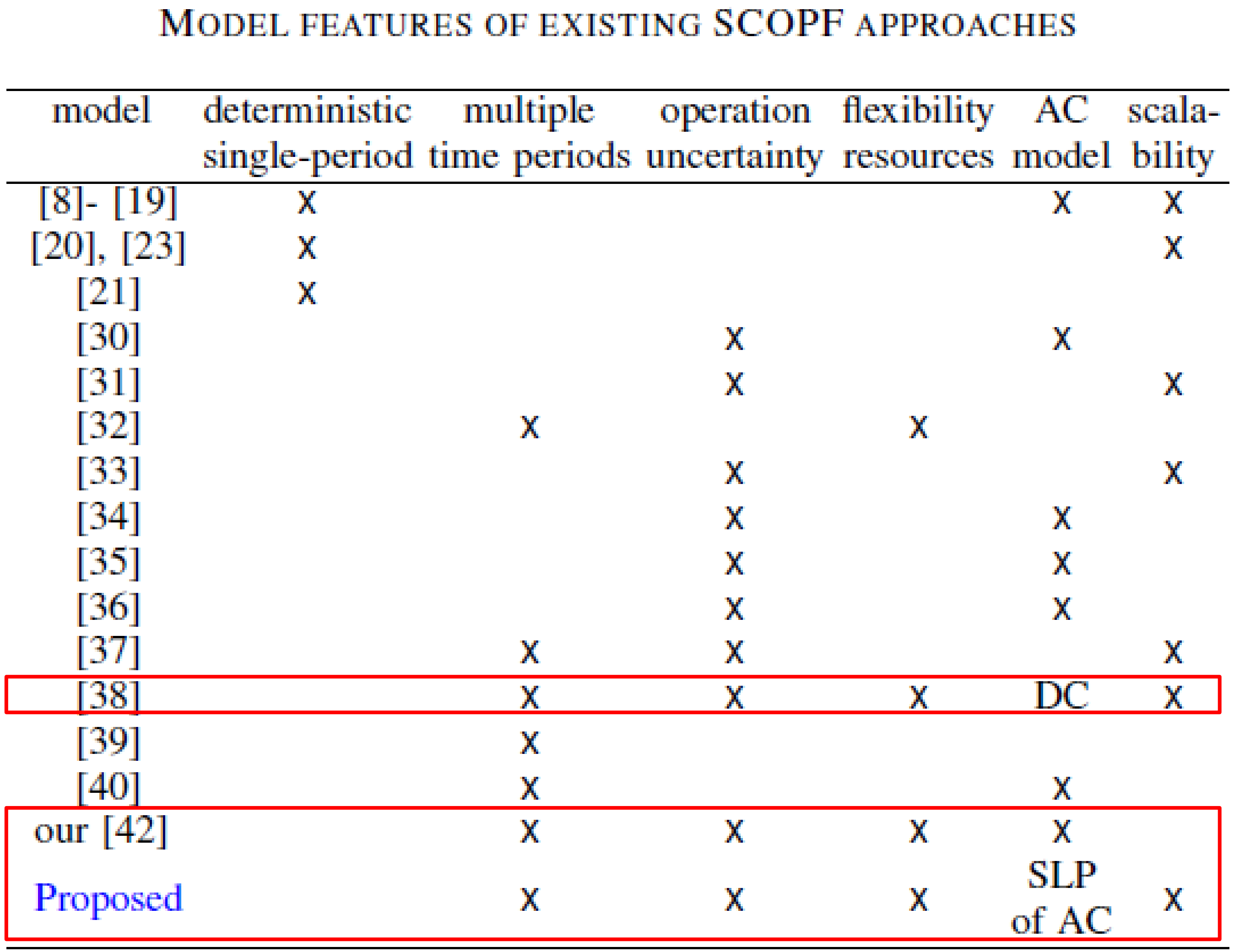
TSO METHODOLOGY PRINCIPLES

- A novel tractable solution methodology, which integrates:
  - A **linear programming** (**LP**) model inspired from [R]
    - Model linearized in the space of V^2 and θ variables
    - DC model augmented with voltage dependent terms and nodal reactive power balance equation
  - N-1 security assessment based on AC power flow
  - ARIMA models
  - speed-up techniques (constraints selection and tightening)

- The linear model from [R] (for single-period deterministic AC-OPF) was extend significantly by considering jointly:
  - contingencies
  - uncertainty scenarios
  - multiple time periods
  - emerging flexible options
  - here and now and wait and see decisions
  - as well as validate it numerically in various and stressed operating conditions

[R] Z. Yang, H. Zhong, A. Bose, T. Zheng, Q. Xia, C. Kang, A linearized OPF model with reactive power and voltage magnitude: A pathway to improve the MW-only DC OPF, IEEE Transactions on Power Systems, vol. 33, no. 2, pp. 1734–1745, 2017.

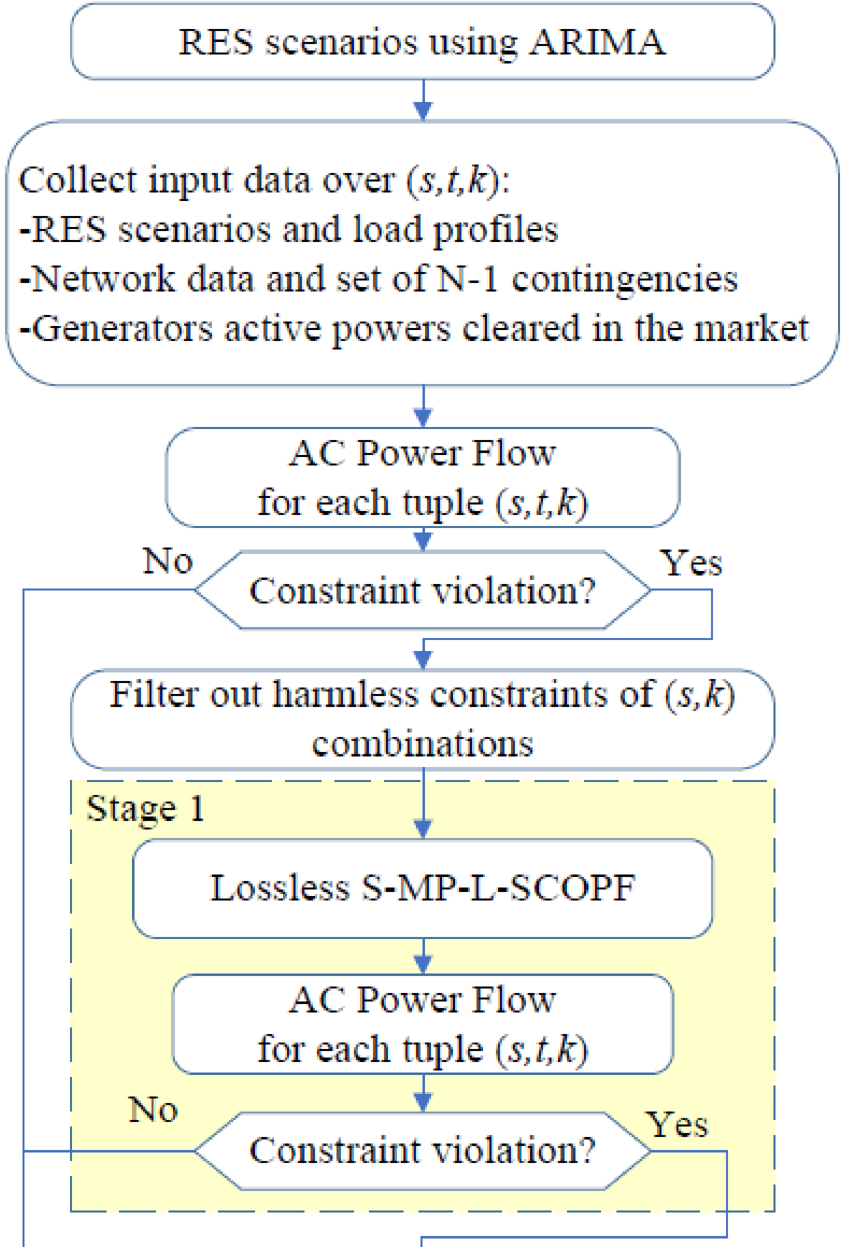
SALIENT FEATURES



MODEL FEATURES OF EXISTING SCOPF APPROACHES

model	deterministic single-period	multiple time periods	operation uncertainty	flexibility resources	AC model	scala-bility
[8]- [19]	x				x	x
[20], [23]	x					x
[21]	x					
[30]			x		x	
[31]			x			x
[32]		x		x		
[33]			x			x
[34]			x		x	
[35]			x		x	
[36]			x		x	
[37]		x	x			x
[38]		x	x	x	DC	x
[39]		x				
[40]		x			x	
our [42]		x	x	x	x	
Proposed		x	x	x	SLP of AC	x

SEQUENTIAL LINEARIZATION ALGORITHM



RES scenarios using ARIMA

Collect input data over (s,t,k):
-RES scenarios and load profiles
-Network data and set of N-1 contingencies
-Generators active powers cleared in the market

AC Power Flow
for each tuple (s,t,k)

Constraint violation?

Filter out harmless constraints of (s,k) combinations

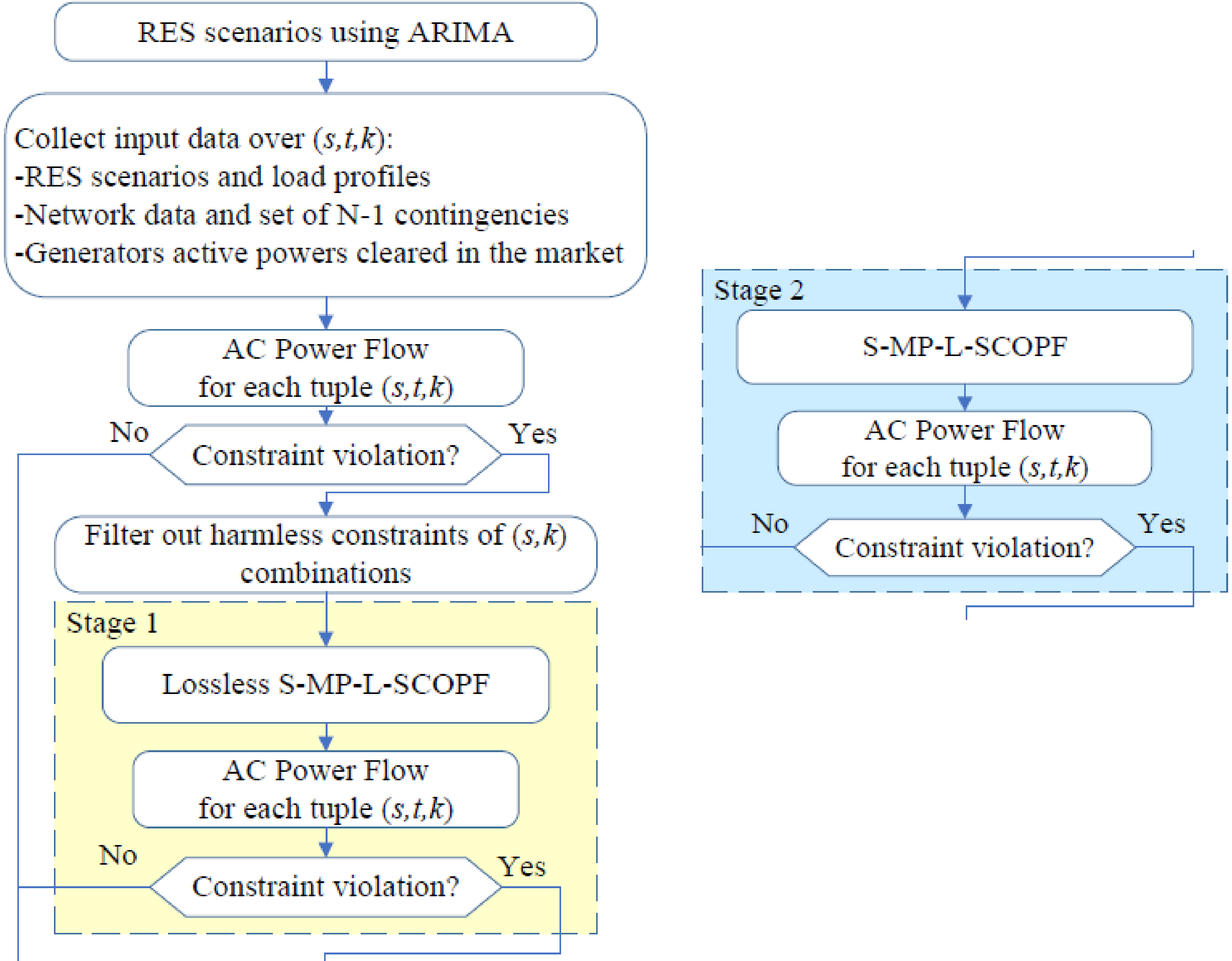
Stage 1

Lossless S-MP-L-SCOPF

AC Power Flow
for each tuple (s,t,k)

Constraint violation?

SEQUENTIAL LINEARIZATION ALGORITHM



RES scenarios using ARIMA

Collect input data over (s,t,k):
-RES scenarios and load profiles
-Network data and set of N-1 contingencies
-Generators active powers cleared in the market

AC Power Flow for each tuple (s,t,k)

Constraint violation?

No

Yes

Filter out harmless constraints of (s,k) combinations

Stage 1

Lossless S-MP-L-SCOPF

AC Power Flow for each tuple (s,t,k)

Constraint violation?

No

Yes

Stage 2

S-MP-L-SCOPF

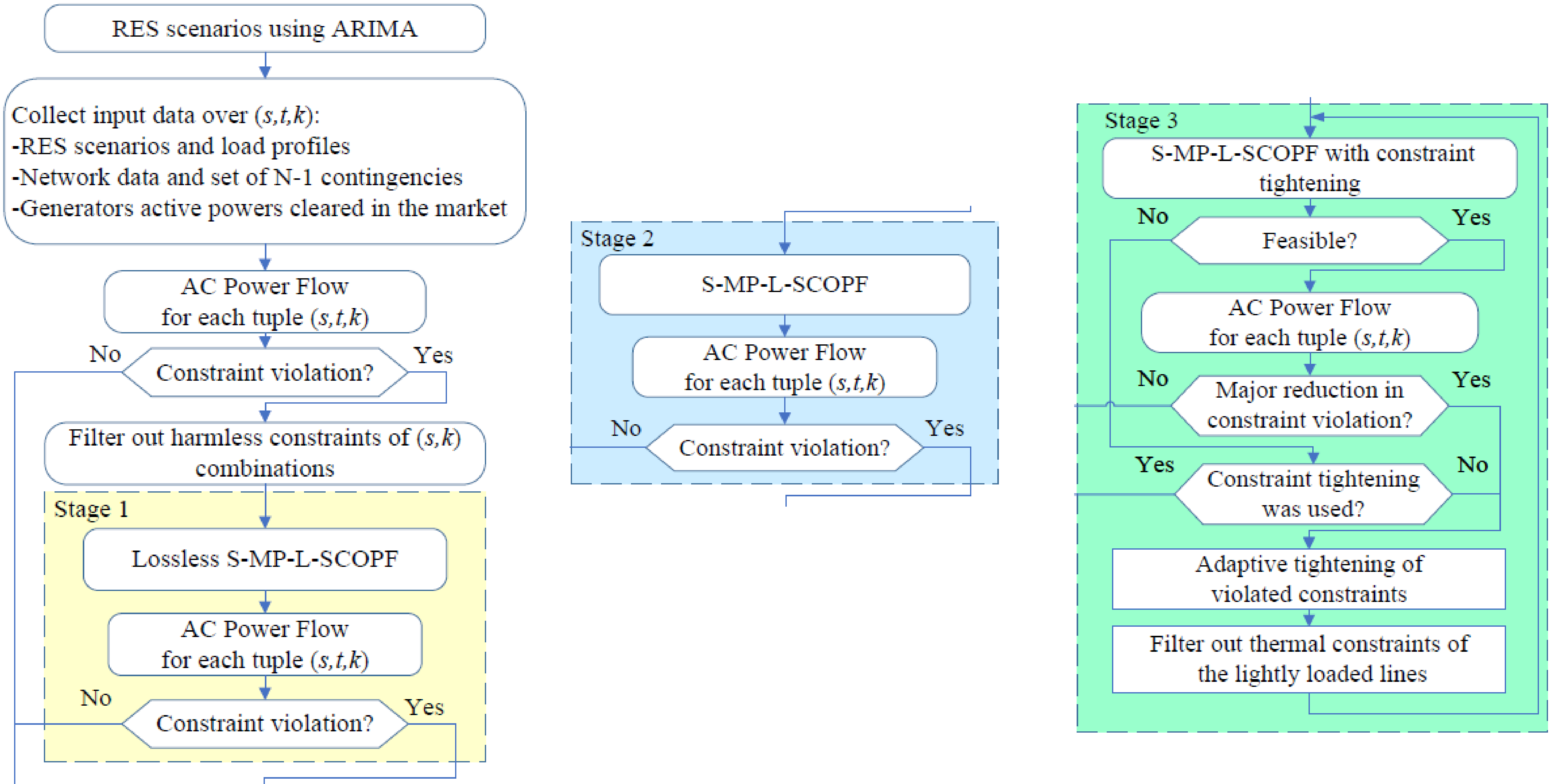
AC Power Flow for each tuple (s,t,k)

Constraint violation?

No

Yes

SEQUENTIAL LINEARIZATION ALGORITHM



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Lossless S-MP-L-SCOPF

AC Power Flow for each tuple (s,t,k)

Constraint violation?

Stage 2

S-MP-L-SCOPF

AC Power Flow for each tuple (s,t,k)

Constraint violation?

Stage 3

S-MP-L-SCOPF with constraint tightening

Feasible?

AC Power Flow for each tuple (s,t,k)

Major reduction in constraint violation?

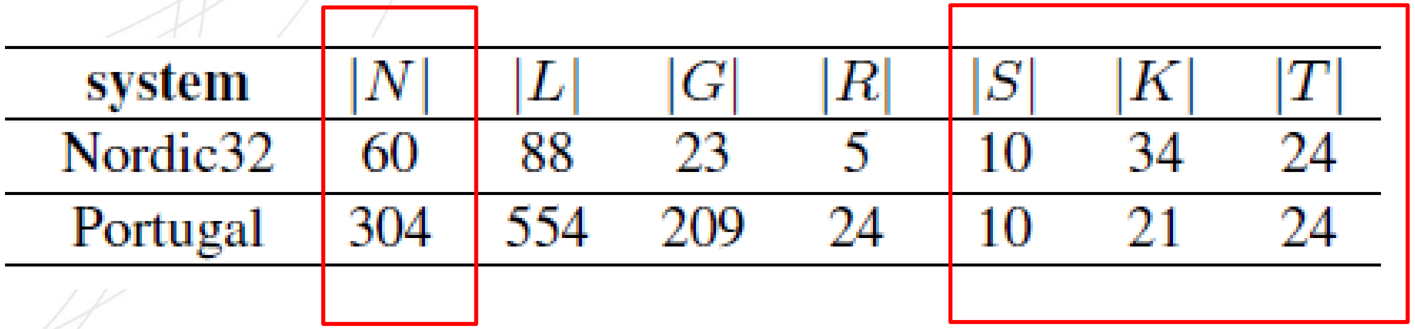
Constraint tightening was used?

Adaptive tightening of violated constraints

Filter out thermal constraints of the lightly loaded lines

CASE STUDIES

- Proposed tractable approach is evaluated on the two transmission grids



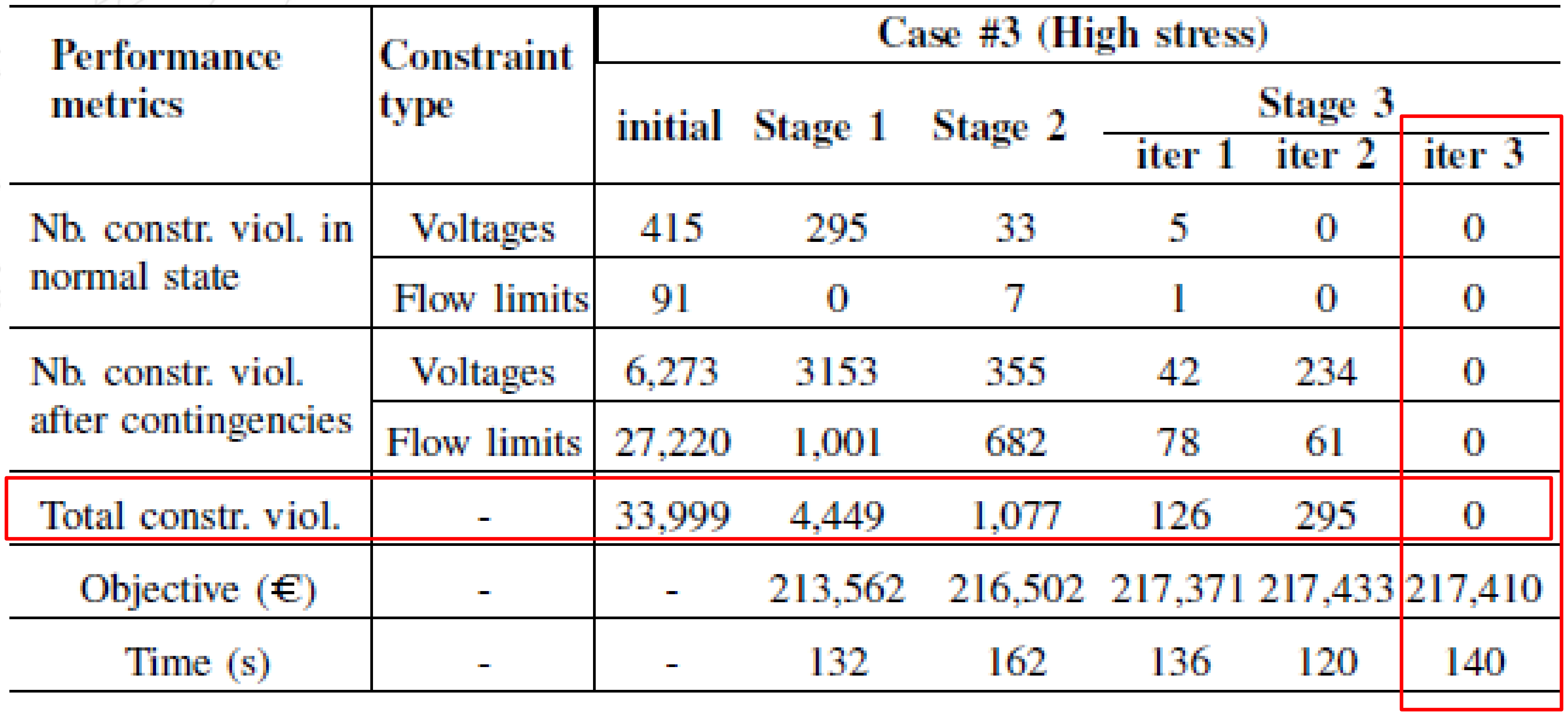
system	$\|N\|$	$\|L\|$	$\|G\|$	$\|R\|$	$\|S\|$	$\|K\|$	$\|T\|$
Nordic32	60	88	23	5	10	34	24
Portugal	304	554	209	24	10	21	24

- 2 storages and 3 flexible loads

- **24** time periods and **10** RES uncertainty scenarios (generated using ARIMA)

- All networks provide flexibility in the form of:
  - Active power curtailment of RES, reactive power provision from RES
  - Active power charging/discharging of an electrical energy storage
  - Active power over/under-demand of a flexible load

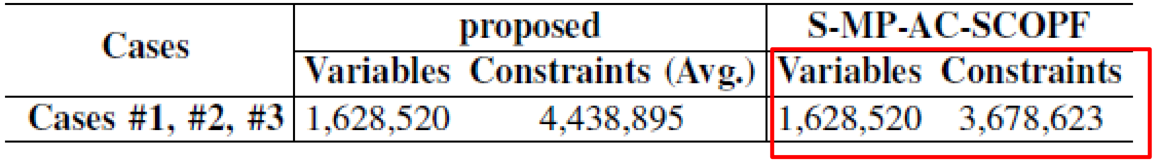
CASE STUDY: NORDIC32 SYSTEM



Performance metrics	Constraint type	Case #3 (High stress)					
		initial	Stage 1	Stage 2	Stage 3		
					iter 1	iter 2	iter 3
Nb. constr. viol. in normal state	Voltages	415	295	33	5	0	0
	Flow limits	91	0	7	1	0	0
Nb. constr. viol. after contingencies	Voltages	6,273	3153	355	42	234	0
	Flow limits	27,220	1,001	682	78	61	0
Total constr. viol.	-	33,999	4,449	1,077	126	295	0
Objective (€)	-	-	213,562	216,502	217,371	217,433	217,410
Time (s)	-	-	132	162	136	120	140

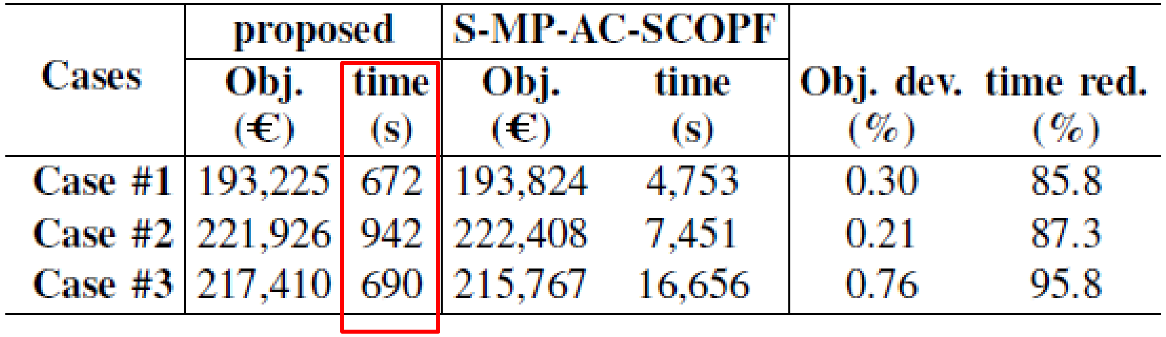
NORDIC32 SYSTEM: COMPARISON WITH IPOPT

IPOPT



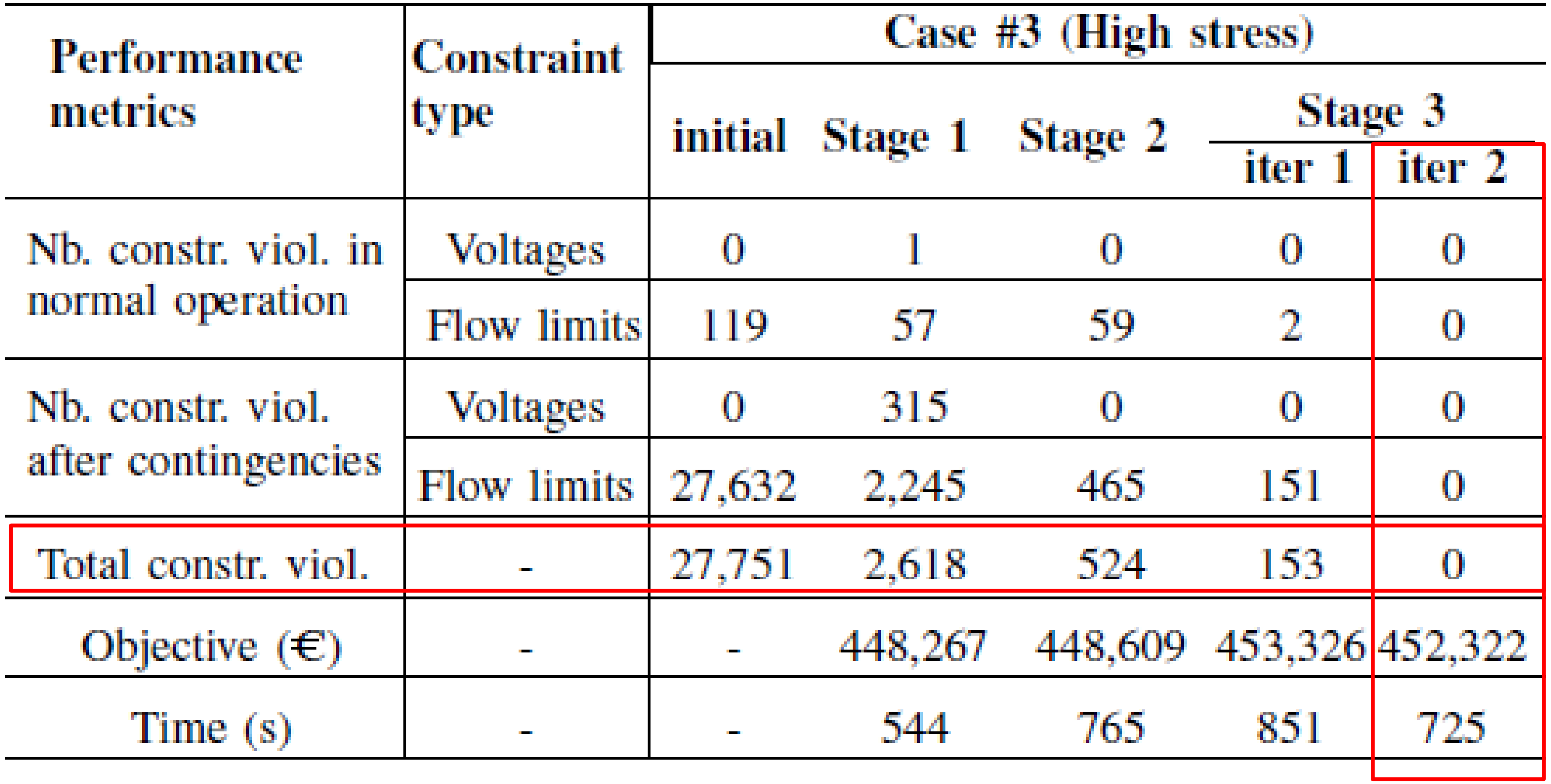
Cases	proposed		S-MP-AC-SCOPF	
	Variables	Constraints (Avg.)	Variables	Constraints
Cases #1, #2, #3	1,628,520	4,438,895	1,628,520	3,678,623

IPOPT



Cases	proposed		S-MP-AC-SCOPF			
	Obj. (€)	time (s)	Obj. (€)	time (s)	Obj. dev. (%)	time red. (%)
Case #1	193,225	672	193,824	4,753	0.30	85.8
Case #2	221,926	942	222,408	7,451	0.21	87.3
Case #3	217,410	690	215,767	16,656	0.76	95.8

CASE STUDY: PORTUGUESE SYSTEM

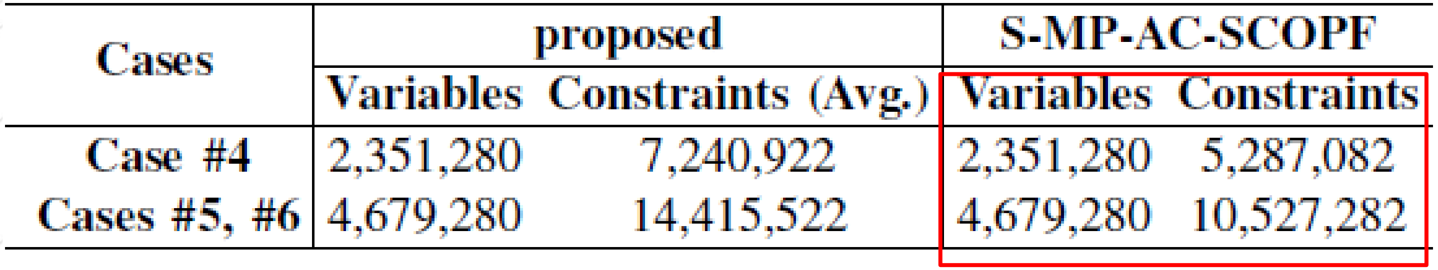


Performance metrics	Constraint type	Case #3 (High stress)				
		initial	Stage 1	Stage 2	Stage 3	
					iter 1	iter 2
Nb. constr. viol. in normal operation	Voltages	0	1	0	0	0
	Flow limits	119	57	59	2	0
Nb. constr. viol. after contingencies	Voltages	0	315	0	0	0
	Flow limits	27,632	2,245	465	151	0
Total constr. viol.	-	27,751	2,618	524	153	0
Objective (€)	-	-	448,267	448,609	453,326	452,322
Time (s)	-	-	544	765	851	725

- The methodology is able to reduce progressively, at each stage or iteration, the number and magnitude of thermal or voltage violated constraints until reaching feasibility.

PORTUGUESE SYSTEM: COMPARISON WITH IPOPT

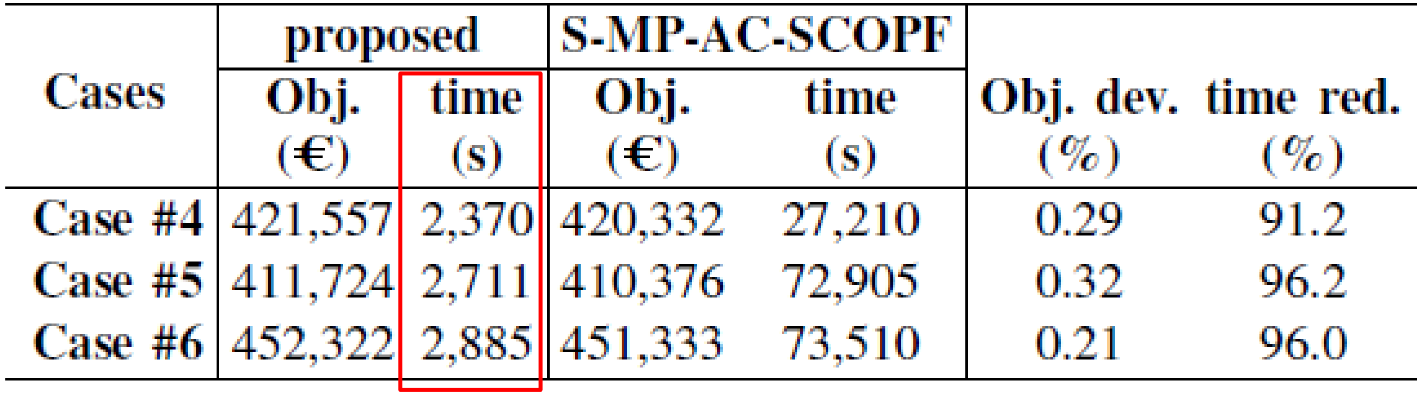
IPOPT



Cases	proposed		S-MP-AC-SCOPF	
	Variables	Constraints (Avg.)	Variables	Constraints
Case #4	2,351,280	7,240,922	2,351,280	5,287,082
Cases #5, #6	4,679,280	14,415,522	4,679,280	10,527,282

2 times more scenarios

IPOPT



Cases	proposed		S-MP-AC-SCOPF			
	Obj. (€)	time (s)	Obj. (€)	time (s)	Obj. dev. (%)	time red. (%)
Case #4	421,557	2,370	420,332	27,210	0.29	91.2
Case #5	411,724	2,711	410,376	72,905	0.32	96.2
Case #6	452,322	2,885	451,333	73,510	0.21	96.0

CONCLUSIONS

- **For the first-time** the most complete and challenging uncertainty-aware and flexibility-driven SCOPF problem to date, the **S-MP-SCOPF, is solved with acceptable precision and speed**.

- Extensive results demonstrate **empirically** (we don't have a mathematical proof of convergence!) that:

- regarding **solution accuracy**, the proposed methodology provides a **near-optimal** and **feasible** solution:
  - The optimality gap is small (0.21% to 0.76%) in all test cases
  - AC-feasible solution has been reached after solving **four to five large LP problems**

- regarding **computational efficiency**, the proposed methodology **is 8-10 times faster than IPOPT** solver for NLP
  - The methodology outperforms alternative linear models
  - It takes overall 15 minutes for the Nordic system and **45 minutes for the Portuguese system**

- Further work: scalability to larger systems via decomposition and **calculations parallelization**
  - waiting for quantum computers ☺

FOR FURTHER READING AND CODE SHARING

- Problem formulation:
  - [1] M.I. Alizadeh, M. Usman, F. Capitanescu, Envisioning security control in renewable dominated power systems through stochastic multi-period AC security constrained optimal power flow, International Journal of Electrical Power & Energy Systems 139, 107992, 2022.

- Solution algorithm:
  - [2] M.I. Alizadeh, F. Capitanescu, A Tractable Linearization-Based Approximated Solution Methodology to Stochastic Multi-Period AC Security-Constrained Optimal Power Flow, IEEE Transactions on Power Systems, in press, 2022.

The code source of [2] will be open this early next year on ATTEST repository: stay tunned!

Thank you for your attention!

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