Academia versus real-world in optimizing power system operation: the case of security-constrained optimal power flow

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Outline of the presentation

- Gentle introduction to optimization
- Academic research: what benefit for the society?
- Gaps in optimal power flow (OPF)
  - Solution methods (local optimizers vs convex relaxations)
  - Some further complexity related to the OPF problem
  - Suppressing ineffective control actions in OPF
  - Handling of discrete variables
- Gaps in security constrained optimal power flow (SCOPF)
  - Methodology to reduce the huge problem size
  - Multiple limits in post-contingency state
  - The use of a limited number of corrective actions
  - Modeling of corrective actions based on TSO operation rules
  - Usable solutions for infeasible problems
- Further key needs and conclusions

Gentle high-level introduction to optimization

Mathematical optimization and ... types of doctors

associate any health issue to a specific doctor (specialist)

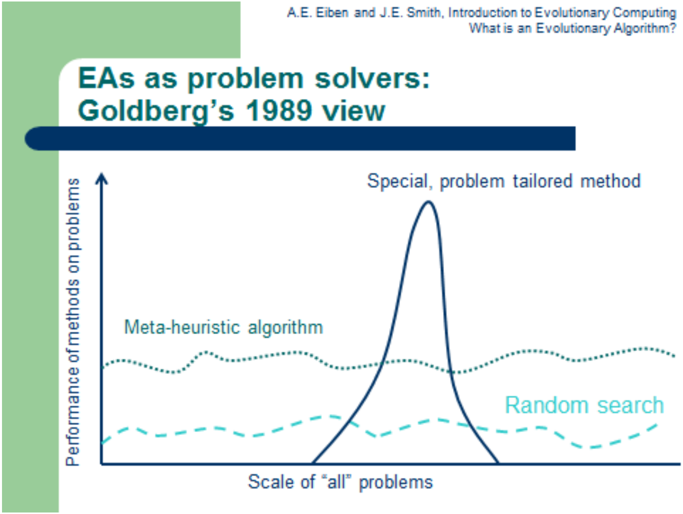
Types (classes) of optimization problems

- an optimization problem is defined by the **triple**
  - objective function, decision variables, constraints
- solving **efficiently** optimization problems requires **tailored** algorithms
  - objective/constraints:
    - linear versus nonlinear
    - convex versus non-nonvex
    - continuous versus discountinuous
  - decision variables:
    - continuous versus binary/discrete
  - additional potential features:
    - no objective (feasibility only) $\rightarrow$ constraint programming
    - complementarity (equilibrium) constraints
    - deterministic versus **uncertainty-infused** problems (stochastic, robust, chance constrained)
    - single objective versus multi-objective problems (Pareto front)
    - (intricate) multi-level optimization problem (e.g. bi-level)

How to solve an optimization problem?

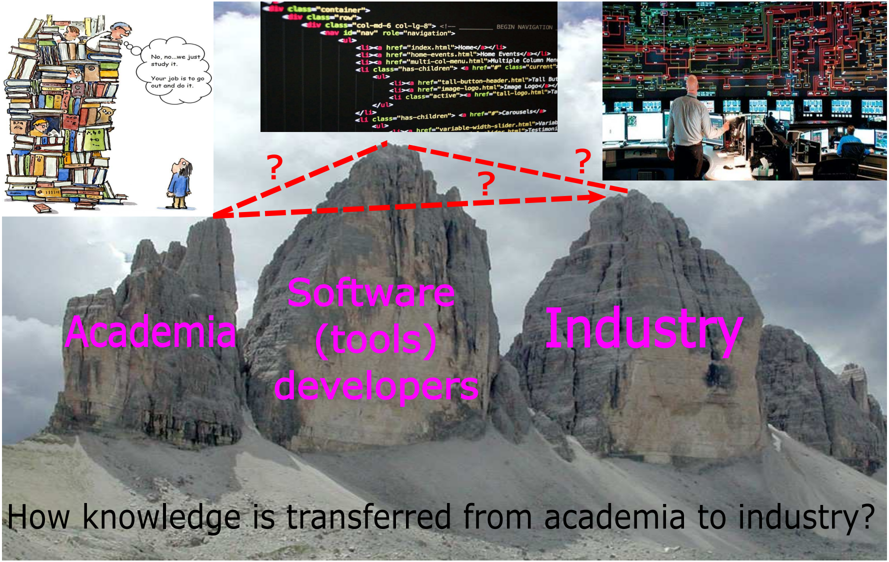
- formulate the optimization problem in such a way such that to exploit its structure and features
    - often using smart reformulation to an **equivalent** problem
- *tune* and use **generic** off the shelf solvers
- ... even better develop a **tailored** algorithm if generic solver performance is not satisfactory

Choice of optimizers: general purpose vs tailored



Optimization approach: academia versus real-world

Bridging the *death valley* from academia to industry



Criticism from industry folks

- Academia is ...
  - working on (mathematically clean) over-simplified OPF problems
  - **developing solutions in search of a problem**

Criticism from industry folks

- Academia is ...
  - working on (mathematically clean) over-simplified OPF problems
  - **developing solutions in search of a problem**
- Feedback from industry:
  - needs and requirements of OPF tools spelled out
  - large scale synthetic data sets (e.g. supplied with MATPOWER and others): thanks to RTE France initiative
  - Grid Optimization Competition of US Department of Energy ARPA-E https://gocompetition.energy.gov/
    - development of disruptive methods for **preventive** SCOPF
    - good progress but still missing key aspects (corrective mode, discrete variables, decision variables other than generators)!
    - I. Avramidis et al." A novel approximation of SCOPF with incorporation of generator frequency and voltage control response", IEEE TPWRS, 2021
- On the other hand ... **realistic** testbed OPF problems (data and full formulation) are non-existent!

OPF/SCOPF: problem formulation background

- **1962**: J. Carpentier:
  formulation of the OPF problem
  targeting economic operation of a power system
- **1974**: O. Alsac and B. Stott:
  formulation/solution of the SCOPF in *preventive* mode
- **1987**: A. Monticelli, M. Pereira, S. Granville:
  formulation/solution of the SCOPF in *corrective* mode
- **2012**: F. Capitanescu, S. Fliscounakis, P. Panciatici, L. Wehenkel:
  solution of the SCOPF *under uncertainties*

Conventional (deterministic) OPF formulation

non-convex nonlinear programming (NLP) problem

$$\min_{\mathbf{x},\mathbf{u}} f(\mathbf{x},\mathbf{u}) \quad \leftarrow \text{objective function: generation cost}$$

$$\text{s.t. } \mathbf{g}(\mathbf{x},\mathbf{u}) = \mathbf{0} \quad \leftarrow \text{AC power flow equations}$$

$$\mathbf{h}(\mathbf{x},\mathbf{u}) \leq \mathbf{0} \quad \leftarrow \text{operation limits: currents, voltages}$$

$$\underline{\mathbf{u}} \leq \mathbf{u} \leq \overline{\mathbf{u}} \quad \leftarrow \text{physical limits of control variables}$$

- $\mathbf{x}$ - state/dependent variables:
  magnitude V and angle θ of complex voltage at all buses
- $\mathbf{u}$ - continuous control/independent variables:
  active and reactive powers of generators

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$$

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Is the solution of this problem an industry need? **NO!**
... it is a (simplified) buliding block in SCOPF

Solution methods for the NLP core optimizer

trade-off: optimality vs **reliability** vs **speed**

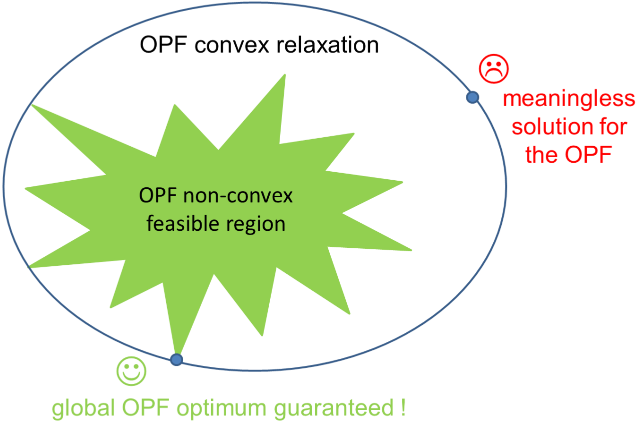
local optimizers: (at least) local optimum solution

- 1968: gradient method (H. Dommel and W. Tinney)
- 1973: sequential linear programming (O. Alsac and B. Stott)
- 1973: sequential quadratic programming (G. Reid and L. Hasdorf)
- 1984: Newton method (D. Sun et al.)
- 1994: interior-point method (Y. Wu et al., and S. Granville)

global optimizers: global optimum of a RELAXED convex problem

- 2012: convex relaxation (semidefinite programming) (J. Lavaei and S. Low)

Convex relaxations rationale



Convex relaxations: pros, cons, main findings

- provides a (tight?) lower bound on the NLP problem optimum
- if the duality gap of the convex relaxed problem is zero then its solution is also the global optimum of the original problem
  - else: convex relaxation solution is not physically meaningful!
- provides a certificate of problem infeasibility

Convex relaxations: pros, cons, main findings

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  - else: convex relaxation solution is not physically meaningful!
- provides a certificate of problem infeasibility
- the solution obtained with a local optimizer is the global optimum (or a solution of very high quality) in most cases
- in the vast majority of experiments the relaxation did not return a feasible solution to the original non-convex problem!
- scalability remains to be proven (despite theoretical guarantees)
- phylosophical question: one does really need the global optimum of core NLP of MINLP problems?

Further complexity of OPF problems

OPF dispatch: active power vs. reactive power

Under **normal operating conditions** generally:

- active power flows are weakly coupled with voltage magn. V
- reactive power flows are weakly coupled with voltage angles θ

	active power	**reactive power**
control variables	generator active power phase shifter angle MW scheduled transfers	generator terminal voltage transformer ratio shunt reactor/capacitor
control variables	network topology load curtailment generator start-up/shut-down	network topology load curtailment generator start-up/shut-down
constraints	branch current active power flows	voltage limits reactive power flows
objective function	min generation cost min controls deviation	min power losses max reactive power reserves

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Challenges to the OPF problem

- Suppressing ineffective control actions
- Handling of discrete variables

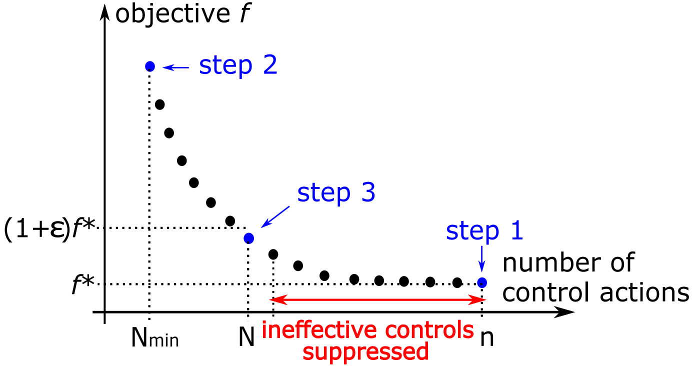
Why suppressing ineffective control actions in OPF?

- all control actions proposed by an OPF are truly effective to an operator?
- system operators want to understand why each control action proposed by OPF is needed and if it matches their experience
- issues for conventional OPF:
  - most conventional OPFs use the whole set of controls to solve the problem and very often (almost) all of them have moved at the optimal solution (some to rather **arbitrary values**)
  - almost every control variable participates in a non separable way to improving the objective and satisfying the constraints
  - control actions are not easy to rank and the effectiveness of an action is not necessarily related to its magnitude
- large but inefficient redispatch on some decision variables!

Why suppressing ineffective control actions in OPF?

- OPF input data are not perfectly known or (slightly) noisy
- the OPF problem model is an approximation of the reality fed with (slightly) imperfect data
- the rationale:
  - only effective control actions of an OPF should be computed as they have high likelihood to remain efficient in practice
  - the effect in practice of implementing also ineffective OPF control actions **may be offset** by the imperfect data/model
- meaning of optimization in practice is
  - improvement of operation performance of slightly noisy or imperfectly known real world system models
  - NOT rigorous optimization of academic ideal models
- suppressing ineffective control actions is important

The concept of suppressing ineffective controls



F. Capitanescu, Suppressing ineffective control actions in optimal power flow problems, IET GTD 14 (13), 2520-2527, 2020

- n - the number of available controls in conventional OPF
- $N_{\min}$ - the minimal number of controls to ensure feasibility
- N - the number of effective control actions

OPF problem formulation as a **MINLP**

$$
\begin{aligned}
\min_{\mathbf{x},\mathbf{u}_c,\mathbf{u}_d} \quad & f(\mathbf{x},\mathbf{u}_c,\mathbf{u}_d) && \leftarrow \text{generation cost, power losses} \\
\text{s.t.} \quad & \mathbf{g}(\mathbf{x},\mathbf{u}_c,\mathbf{u}_d) = \mathbf{0} && \leftarrow \text{AC power flow equations} \\
& \mathbf{h}(\mathbf{x},\mathbf{u}_c,\mathbf{u}_d) \leq \mathbf{0} && \leftarrow \text{operational limits on I, P, S, V} \\
& \underline{\mathbf{u}}_c \leq \mathbf{u}_c \leq \overline{\mathbf{u}}_c && \leftarrow \text{bounds on control variables} \\
& \mathbf{u}_d = [u_{d1} \dots u_{di} \dots u_{dn_d}]^T && \\
& u_{di} \in \{u_{di}^1, \dots, u_{di}^j, \dots, u_{di}^{p(i)}\} && \leftarrow \text{discrete variables values}
\end{aligned}
$$

- $\mathbf{x}$ - state/dependent variables:
  magnitude V and angle θ of complex voltage at all buses
- $\mathbf{u}_c$ - continuous control variables:
  generator active power, generator terminal voltage, etc.
- $\mathbf{u}_d$ - discrete/binary control variables:
  transformer ratio, phase shifter angle, shunt reactive power (capacitors/reactors), network topology, etc.

Handling of discrete variables

- Variables with small discrete steps:
  transformer ratio
  phase shifter angle
  shunt reactors/capacitors reactive power
- Variables with large discrete steps and binary variables:
  network switching
  unit start-up/shut down
  shunt reactors/capacitors reactive power

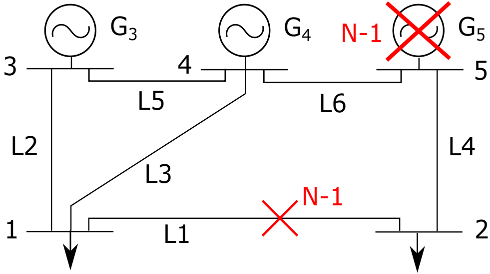
Handling of discrete variables

- OPF is a mixed-integer nonlinear programming problem
  - MINLP classical methods are not yet sufficiently mature to cope with very large size (non-convex) problems
- handling of discrete variables is a **trade-off** between:
  - degree of sub-optimality
  - reliability (ability to deal with infeasible discrete variables configurations)
  - computational speed
- related works:
  - simple heuristics: round-off, progressive round-off
  - penalty functions within NLP or LP solvers
  - ordinal optimization
  - mixed-integer linear programming (MILP)
  - interior point cutting plane
  - global optimization methods:
    genetic algorithms, simulate annealing, tabu search

Biggest gap: handling N-1 security is the core business of TSOs!
Conventional Security Constrained Optimal Power Flow (SCOPF)

What is power system security?

- a **contingency** is the unexpected disconnection of one or multiple system elements (e.g. generator, line)
- **security** = power system ability to **withstand** contingencies
  - ensure a **stable** transition towards a **viable equilibrium point** without loss of load
- power system operation must comply with the **N-1 security criterion**
  - that is at any time the system must be able to withstand the loss of any **single** equipment



Day-ahead operational planning

- **SCOPF**: computes cost-optimal preventive/corrective control actions to satisfy **static security constraints** (thermal & voltages) for each foreseen operation state of next day

Conventional (deterministic) SCOPF formulation

$$
\begin{aligned}
\min_{\mathbf{x}_0,\ldots,\mathbf{x}_c,\mathbf{u}_0,\ldots,\mathbf{u}_c} \quad & f(\mathbf{x}_0, \mathbf{u}_0) & & \\
\text{s.t.} \quad & \mathbf{g}_0(\mathbf{x}_0, \mathbf{u}_0) = \mathbf{0} & & \leftarrow \text{ base case constraints} \\
& \mathbf{h}_0(\mathbf{x}_0, \mathbf{u}_0) \le \mathbf{0} & & \leftarrow \text{ base case constraints} \\
& \mathbf{g}_k(\mathbf{x}_k, \mathbf{u}_k) = \mathbf{0} & k = 1, \ldots, c & \leftarrow \text{ contingency } k \text{ constraints} \\
& \mathbf{h}_k(\mathbf{x}_k, \mathbf{u}_k) \le \mathbf{0} & k = 1, \ldots, c & \leftarrow \text{ contingency } k \text{ constraints} \\
& |\mathbf{u}_k - \mathbf{u}_0| \le \Delta \mathbf{u}_k^{max} & k = 1, \ldots, c & \leftarrow \text{ ``coupling'' constraints}
\end{aligned}
$$

- **x** - state/dependent variables:
  magnitude V and angle θ of complex voltage at all buses
- **u** - continuous and discrete control variables:
  generator active power, terminal voltage, transformer ratio, phase shifter angle, shunt capacitors/reactors reactive power

Features and challenges of the SCOPF problem

- *nonlinear:* includes power flow equations and other nonlinear inequality constraints
- **non-convex:** includes power flow equations and bounds on other nonlinear inequality constraints
- *with continuous* and **discrete variables**
- *static:* refers to a single operating point in time
- **large scale:** the SCOPF problem for a 3000-bus system and 999 contingencies contains:
  around 2000 x 3000 = 6.000.000 equality constraints
  around 6000 x 3000 = 18.000.000 inequality constraints
  around 1000 x 3000 = 3.000.000 control variables

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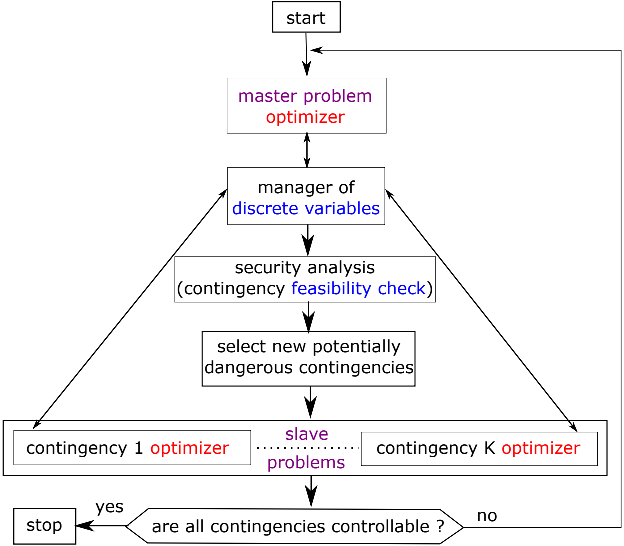
  around 1000 x 3000 = 3.000.000 control variables
- **academia** *simplifies* **SCOPF to a large scale MINLP**
- intractable on a normal computer due to memory limitation!
- *scalable decomposition* is essential as a limited number of constraints are binding

SCOPF decomposition methodology



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Numerical results with ULg-GDF Suez methodology

- coded mainly by Dr. Ludovic Platbrood in EU-FP7 PEGASE
- model the whole European transmission system
- **9241** buses (**5000** control variables) and **12000** contingencies
- HPC: BladeCenter, 8 blades, 8 cores per blade, 2.6 Ghz clock rate
- overall time (with **from the scratch** assumptions): **65 minutes**

				computation time (s)		
iteration	variables	constraints	cont	core optimizer	security analysis	network compression
1	23000	50000	0	70	130	60
2	30000	64000	23	485	130	140
3	33000	70000	37	940	130	140
4	34000	72000	40	710	130	0
				2205 **57 %**	520 13 %	340 9 %

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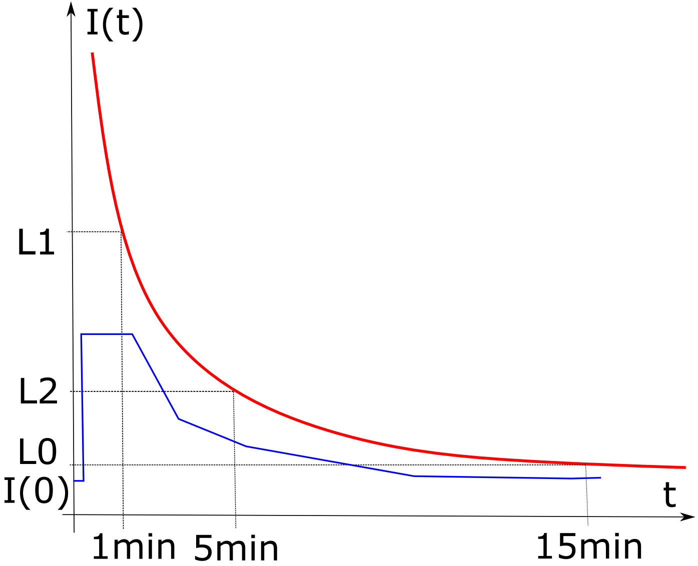
Further modelling issues in SCOPF

Multiple post-contingency line thermal limits



Limiting the number of corrective actions in SCOPF

$$
\begin{aligned}
& -\mathbf{s}_k \Delta \mathbf{u}_k^{\max} \leq \mathbf{u}_k - \mathbf{u}_0 \leq \mathbf{s}_k \Delta \mathbf{u}_k^{\max} && k = 1, \ldots, c \\
& \mathbf{1}^T \mathbf{s}_k \leq N_k && k = 1, \ldots, c \\
& \mathbf{s}_k \in \{0, 1\} && k = 1, \ldots, c
\end{aligned}
$$

- N_k is the maximum number of corrective actions allowed
- $\mathbf{s}_k$ is a vector of statuses of corrective actions
  - if $s_{kj} = 1$ then the corrective action u_{kj} is allowed: $-\Delta u_{kj}^{\max} \leq u_{kj} - u_{0j} \leq \Delta u_{kj}^{\max}$
  - if $s_{kj} = 0$ then this action is not allowed: $u_{kj} = u_{0j}$
- binary variables $\mathbf{s}_k$ increases the problem complexity
- possible approach for **thermal constraints**: compute $\mathbf{s}_k$ from the MILP problem approximation
  - of the whole SCOPF problem
  - of each post-contingency state (simulated at the OPF solution)

Modelling of corrective actions based on SO operating rules

- the conventional SCOPF does not model SO operating rules which associate a pre-defined set of corrective actions (determined based on the SO' knowledge of the system) with a given post-contingency constraint violation
- such corrective actions are activated only if the constraints are not satisfied by preventive actions
- the set of constraints of corrective actions based on SO operating rules:

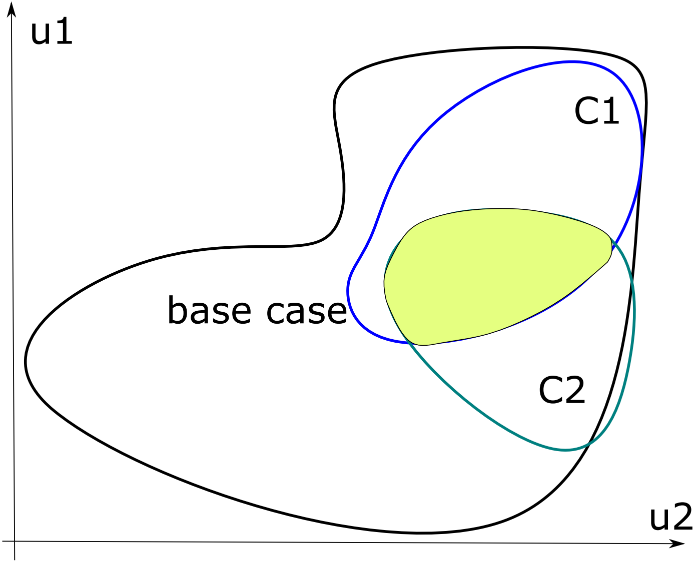
$$\begin{aligned} -\mathbf{b}_k \Delta \mathbf{u}_k^{\max} \leq \mathbf{u}_k - \mathbf{u}_0 \leq \mathbf{b}_k \Delta \mathbf{u}_k^{\max} \quad & k = 1, \ldots, c \\ (\mathbf{b}_k - \mathbf{1})\boldsymbol{\lambda}_k < \mathbf{h}_k(\mathbf{x}_k, \mathbf{u}_k) \leq \mathbf{b}_k \boldsymbol{\lambda}_k \quad & k = 1, \ldots, c \\ \mathbf{b}_k \in \{0, 1\}, \boldsymbol{\lambda}_k > \mathbf{0} \quad & k = 1, \ldots, c \end{aligned}$$

- binary variables $\mathbf{b}_k$ are used to decide the activation of control action $\mathbf{u}_k - \mathbf{u}_0$ $\boldsymbol{\lambda}_k$ is a vector of very large positive values

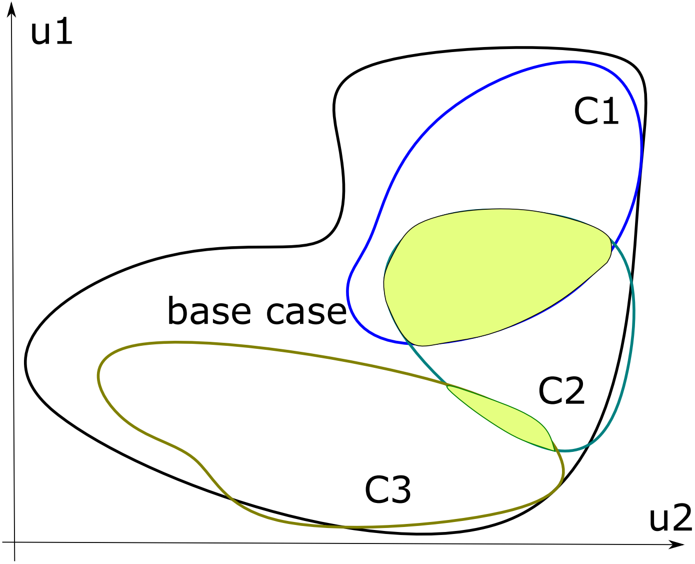
Usable solutions for infeasible SCOPF problems due to conflicting contingencies

F. Capitanescu,
Approaches to Obtain Usable Solutions for Infeasible Security-Constrained Optimal Power Flow Problems Due to Conflicting Contingencies",
IEEE PowerTech, Milano, Italy, 2019.

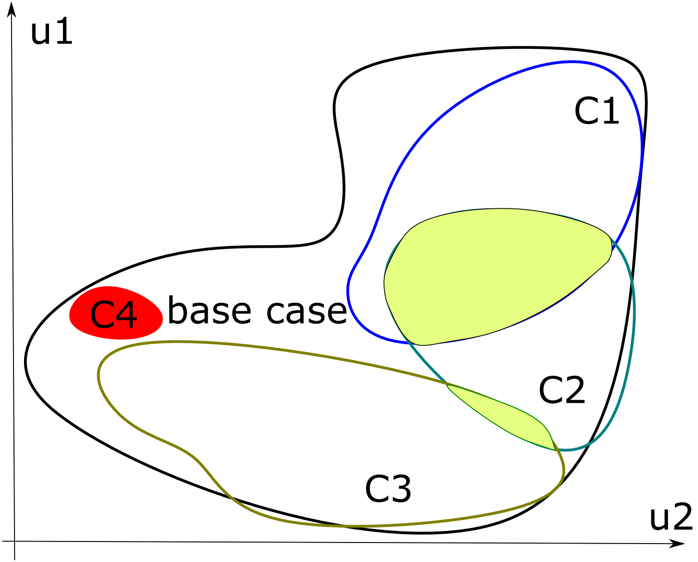
Feasible contingencies



Conflicting contingencies



Conflicting and infeasible contingencies



Relaxation of Control Variables Set

$$
\begin{aligned}
\min_{\mathbf{u}_0,\mathbf{u}_k,\mathbf{s}_k} \; & p_0 f_0(\mathbf{x}_0, \mathbf{u}_0) + \sum_{k \in K} p_k f_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{s}_k) \\
& \mathbf{g}_0(\mathbf{x}_0, \mathbf{u}_0) = \mathbf{0} \\
& \mathbf{h}_0(\mathbf{x}_0, \mathbf{u}_0) \leq \mathbf{0} \\
& \mathbf{g}_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{s}_k) = \mathbf{0} && k \in K \\
& \mathbf{h}_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{s}_k) \leq \mathbf{0} && k \in K \\
& |\mathbf{u}_k - \mathbf{u}_0| \leq \Delta \mathbf{u}_k && k \in K \\
& \mathbf{s}_0 - \mathbf{s}_k \leq \Delta \mathbf{s} && k \in K \\
& \mathbf{1}^T (\mathbf{s}_0 - \mathbf{s}_k) \leq \Delta \mathbf{s}^{\max} && k \in K
\end{aligned}
$$

Relaxation of Operation Constraints Set

$$\min_{\mathbf{u}_0,\mathbf{u}_k,\mathbf{h}_k^+} f_0(\mathbf{x}_0,\mathbf{u}_0) + \beta \sum_{k\in K} \mathbf{h}_k^+$$

$$\begin{aligned}
&\mathbf{g}_0(\mathbf{x}_0,\mathbf{u}_0) = \mathbf{0} && \\
&\mathbf{h}_0(\mathbf{x}_0,\mathbf{u}_0) \leq \mathbf{0} && \\
&\mathbf{g}_k(\mathbf{x}_k,\mathbf{u}_k) = \mathbf{0} && k \in K \\
&\mathbf{h}_k(\mathbf{x}_k,\mathbf{u}_k) \leq \mathbf{h}_k^+ && k \in K \\
&|\mathbf{u}_k - \mathbf{u}_0| \leq \Delta\mathbf{u}_k && k \in K \\
&\mathbf{h}_k^+ \geq \mathbf{0} && k \in K
\end{aligned}$$

Conventional AC SCOPF: conclusions

- major progress on AC SCOPF methodologies reported
- AC SCOPF is computationally demanding
  - but still scalable to large systems and sets of contingencies
  - rely on local optimizers (e.g. KNITRO, IPOPT) for **NLP** core
  - convergence reliability of core optimizers should be improved
- under stringent running time requirements (up to one hour):
  - quality of solution (i.e. sub-optimality gap of the MINLP) **is less important** than feasibility (wrt the contingencies)
  - need fast heuristics for the management of discrete variables

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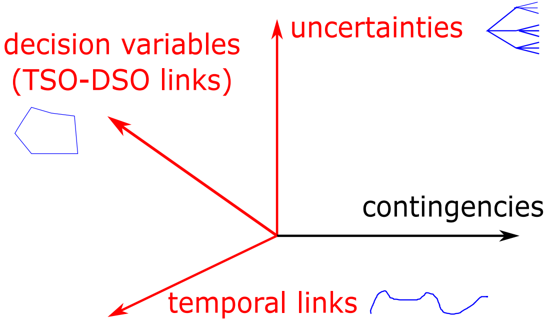
  - need fast heuristics for the management of discrete variables
- TAKE HOME MESSAGE: The OPF community should use the great talent and math inclination to address complex issues beyond the 60 years old Carpentier' formulation!

AC SCOPF: future works

- ... BUT IT DOES NOT FULLY FIT THE TODAY NEED FOR SUSTAINABILITY (I.E. INTEGRATION OF LARGE SHARES OF RENEWABLE GENERATION)!
- trilemma: economics vs security/reliability vs sustainability

AC SCOPF: future works

- ... BUT IT DOES NOT FULLY FIT THE TODAY NEED FOR SUSTAINABILITY (I.E. INTEGRATION OF LARGE SHARES OF RENEWABLE GENERATION)!
- trilemma: economics vs security/reliability vs sustainability
  - expand the SCOPF scope: uncertainty, temporal aspects, TSO-DSO cooperation, etc.



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Advanced formulations/algorithms for SCOPF

- EU Horizon 2020 project ATTEST (03/2020 - 02/2023) https://attest-project.eu/
- ATTEST stands for "Advanced Tools Towards cost-efficient decarbonisation of future reliable Energy SysTems"
- LIST team (Mohammad Iman Alizadeh, Muhammad Usman and myself) develops an advanced formulation/algorithm to extend day-ahead SCOPF to consider (to the best possible extent) uncertainties and time periods
- LIST develops a stochastic multi-period AC SCOPF
  - current capability: 60 nodes, 33 contingencies, 24 periods, 30 uncertainty scenarios $\rightarrow$ equivalent to AC OPF for a 1.5M nodes system
  - research paper under review to EPSR

Conclusions and challenges ahead

- risk-based AC SCOPF and AC SCOPF under uncertainty are in their infancy
- more flexible decision making process balancing risk and uncertainty, adapted to a smart sustainable grid environment
- develop the first generation of tractable risk-based AC SCOPF under uncertainty tools
  - immense potential for new frameworks and scalable algorithms
- improving operation flexibility by shifting the control balance from preventive control to corrective control
- need faster look-ahead SCOPF algorithms close to real time
- extend the risk-based AC SCOPF under uncertainty to:
  - TSO-DSO interfaces (production migrates from TS to DS)
  - multi-periods (to account for energy-based behaviours: demand response, storage)
  - problem size explodes:
    contingencies × uncertainty scenarios × multi-period × DS

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Security management trade-off: uncertainty vs risk